Table S1. Variables and constants used in the mathematical model.

Variable	Value	Units	Reference
P_b	101.3	kPa	1
ρ_{air}	1.20×10^{-3}	mg μl^{-1}	
P_{H_2O}	2.34	kPa	2
$P_{N_2,aq}$	78.12	kPa	2
$P_{O_2,aq}$	20.73	kPa	2
K_{O_2}	6.96×10^{-7}	mm^2 s^{-1} kPa^{-1}	3
K_{N_2}	3.18×10^{-7}	mm^2 s^{-1} kPa^{-1}	3
K_{He}	4.91×10^{-7}	mm^2 s^{-1} kPa^{-1}	3
K_{SF_6}	8.18×10^{-8}	mm^2 s^{-1} kPa^{-1}	3
A	1.87	mm^2	4
a	1.13	mm	4
b	0.42	mm	4
c	0.22	mm	4
M_b	10.18	mg	4
$V_{B, init}$	0.867	μl	5
V_{bs}	9.44	μl	5
$V_{O_2,hb}$	0.25	μl	5
P_{50}	3.90	kPa	6
$P_{O_2,crit}$	2.0	kPa	7
$\dot{V}_{O_2,MR}$	2.17×10^{-3}	μl s^{-1}	4
X	0.04, 0.10, 0.60	mm	3, 1

References: 1, Seymour and Matthews, 2013; 2, Dejours 1981; 3, Rahn and Paganelli, 1968; 4, present study; 5 Matthews and Seymour, 2008; 6, Matthews and Seymour, 2011; 7, Matthews and Seymour 2010. Symbols: P_b = barometric pressure, ρ_{air} = air density, P_g = pressure of relevant gas, K_g = Krogh's coefficient of diffusion of relevant gas, A = surface area for gas exchange, a,b,c = ellipsoid semi-axis values as described in results, M_b = backswimmer body mass, $V_{B, init}$ = initial bubble volume, V_{bs} = backswimmer volume, $V_{O_{2,hb}}$ = initial O_2 volume bound to haemoglobin, P_{50} = P_{O_2} at which haemoglobin is 50% saturated, $P_{O_2,crit}$ = critical P_{O_2}, $\dot{V}_{O_{2,MR}}$ = O_2 consumption rate, X = boundary layer thickness.

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Table S2. Mathematical model used to simulate the decline in backswimmer buoyancy and air store $P\mathrm{O_2}$ during dives. The model has the following assumptions: (1) surface area for gas exchange (*A*) is defined as the surface area of a half-ellipsoid with semi-axes a, b and c, defined in the results and supplementary Table S1, (2) conductance of each gas remains constant throughout each simulation at a level defined by Fick's law as the Krogh's coefficient (K_g) multiplied by the quotient of *A* and the boundary layer thickness (*X*) (Rahn and Paganelli, 1968; Matthews and Seymour, 2010; supplementary material Table S1), (3) the insect is considered to be just below the water's surface where hydrostatic pressure is negligible and is thus not included (Seymour and Matthews, 2013), (4) bubble surface tension is not included due to its minimal impact on total bubble pressure (Seymour and Matthews, 2013), (5) CO_2 production by the insect is excluded as CO_2 diffuses readily into the water surrounding the air store (Ege, 1915; Rahn and Paganelli, 1968), (6) barometric pressure and temperature are constant at 101.3 kPa and 20°C, respectively, (7) water is well-mixed and in equilibrium with atmospheric gas compositions: $P\mathrm{O_2}$ of 20.73 kPa and $P\mathrm{N_2}$ of 78.12 kPa (Dejours, 1981; Rahn and Paganelli, 1968), and (8) initial air store composition considers trace gases as part of the N_2 fraction (Dejours, 1981; Rahn and Paganelli, 1968). The equations relating to the O_2 release from the haemoglobin ($\dot{V}_{\mathrm{O_2,hb}}$), and haemoglobin saturation curves, are outlined in Matthews and Seymour (2011), with values for the P_{50} and O_2 volume stored in haemoglobin ($V_{\mathrm{O_2,hb}}$) shown in Table S1 of the supplementary material. Density of the insect with the bubble was calculated as the sum of bubble mass (= air density × bubble volume, $\rho_{\mathrm{air}} \times V_{\mathrm{b}}$) and body mass ($M_b$), divided by the sum of bubble volume and body volume (V_{bs}) (supplementary material Table S1).

	Equation	*Description*
(1)	$\dot{V}_{\mathrm{g}} = K_{\mathrm{g}} \frac{A}{X} (\Delta P_g)$	Fick's law describes diffusion of N_2 out of the air store and O_2 in, where $\dot{V}_g$ is the rate of change in volume of a particular gas species (μl s^{-1}), K_g is the Krogh's coefficient ($mm^2\ s^{-1}\ kPa^{-1}$), which is the product of capacitance (β; $mm^3\ mm^{-3}\ kPa^{-1}$) and diffusivity (D; $mm^2\ s^{-1}$), *A* is surface area available for gas exchange (mm^2), *X* is thickness of the effective boundary layer (mm), and ΔP_g is the difference between the pressure of the relevant gas species inside and outside the bubble (kPa).
(2)	$V_{\mathrm{b}}(t) = V_{\mathrm{b}}(t-dt) + \left(\dot{V}_{\mathrm{O_2,hb}} + \dot{V}_{\mathrm{O_2,H_2O}} - \dot{V}_{\mathrm{O_2}} - \dot{V}_{\mathrm{N_2,H_2O}}\right) \times dt$	Differential Eq. 2 describes the change in bubble volume (V_b; μl) over time (*t*; s), where $V_{\mathrm{b}}(t)$ is bubble volume at time t, $\dot{V}_{\mathrm{O_2,hb}}$ (μl s^{-1}) is the rate of O_2 release from the haemoglobin (hb), $\dot{V}_{\mathrm{O_2,H_2O}}$(μl s^{-1}) is the

		rate of O_2 diffusion from the surrounding water, $\dot{V}_{O_2}$ (µl s^{-1}) is the O_2 consumption rate by the backswimmer, and $\dot{V}_{N_2,H_2O}$ (µl s^{-1}) is the rate of N_2 loss to the surrounding water.
(3.1) (3.2)	$\dot{V}_{O_2} = P_{O_{2,in}} \times 0.0011$ $\dot{V}_{O_2} = \dot{V}_{O_{2,MR}}$	When air store PO_2 ($P_{O_{2,in}}$; kPa) drops below the critical PO_2 ($PO_{2,crit}$) of 2 kPa, based on measurements from water boatmen (Matthews and Seymour, 2010), the backswimmer's O_2 consumption rate ($\dot{V}_{O_2}$) is dependent on $P_{O_{2,in}}$ (Eq. 5), and is described by Eq. 3.1. The slope of decline in $\dot{V}_{O_{2,MR}}$ (= 0.0011) is the experimentally measured O_2 consumption rate (µl s^{-1}; supplementary material Table S1) from 0 to 2 kPa. However, when $P_{O_{2,in}}$ is greater than the assumed $PO_{2,crit}$, backswimmer $\dot{V}_{O_2}$ is independent of $P_{O_{2,in}}$, and Eq. 3.2 applies.
(4)	$P_{O_2} = \left(\frac{\dot{V}_{O_{2,flux}}}{V_b(t)}\right) \times (P_b - P_{H_2O})$	The incremental change in bubble PO_2 (kPa s^{-1}) is described by Eq. 4, where $\dot{V}_{O_{2,flux}}$ (µl s^{-1}) incorporates the various O_2 fluxes (i.e. $\dot{V}_{O_{2,H_2O}} + \dot{V}_{O_{2,hb}} - \dot{V}_{O_2}$), $V_b(t)$ is bubble volume (µl) at time *t*, P_b is barometric pressure (kPa), and P_{H_2O} is water vapour pressure (kPa).
(5)	$P_{O_{2,in}}(t) = P_{O_{2,in}}(t - dt) + (P_{O_2}) \times dt$	The PO_2 within the air store ($P_{O_{2,in}}$; kPa) is described by a second differential equation.
(6)	$P_{N_2\,in} = P_b - P_{O_2\,in} - P_{H_2O}$	PN_2 within the air store ($P_{N_{2,in}}$; kPa) relates to Dalton's law.
(7)	$A = \left(4\pi\left(\frac{(ab)^{1.6} + (ac)^{1.6} + (bc)^{1.6}}{3}\right)^{\frac{1}{1.6}}\right)/2$	Surface area for gas exchange (*A*, mm^2) is defined as the approximate equation for the surface area of half an ellipsoid with semi-axes a, b and c (mm; defined in supplementary material Table S1).

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