

Fig. S1. Image processing is used to separate and order the horizontal and vertical stripes necessary to achieve 3D reconstruction the flying bird. (A) Cropped version of the original image of projected stripes on the bird captured by the high-speed camera. (B) The original image is first rotated so equally spaced stripes are vertical. There is a depth edge between the bottom of the left wing and the body which is manually separated by coloring it black. (C-D) The Laplacian of a directional Gaussian filter is applied
to image (B). In (C-H) horizontal stripes are identified in the left figures while vertical stripes are identified in the right figures. (E-F) Automated locally adaptive thresholding is applied in order to identify the highest image intensity values (C-D) and only the connected white regions exceeding a minimum directional length requirement are kept. (G-H) Stripes from images (E-F) are ordered in a repeating rainbow pattern using the intersection data between horizontal and vertical stripes. (I) The raw 3D surface data for a single frame. (J) The surface fit to the 3D data.


B


Fig. S2. Automated definition of a bird-fixed reference frame. (A) Surface points are identified that move less than a preset threshold distance in order to filter out the wings. The unit normal of a plane fit to these points is defined as the $z_{b}$ axis. (B) These points are projected onto the fit plane, and the line of symmetry is found and defined as the $x_{b}$ axis. The $y_{b}$ axis is defined as the cross product of the $z_{b}$ and $x_{b}$ axes. These axes directions are held constant as a function of time. (C) Points on the head of the bird are used to find the origin of the reference frame by fitting a paraboloid to these data points and finding the top point. (D) The final body reference frame only translates as a function of time and does so linearly.


Fig. S3. Three plots from Fig. 3 are shown for each of $\mathbf{4}$ downstrokes. (A-C) The first downstroke (41\% to $65 \%$ of the downstroke) after takeoff in video 1. (D-F) The second downstroke ( $36 \%$ to $64 \%$ of the downstroke) after takeoff in video 1. (G-I) The first downstroke ( $41 \%$ to $65 \%$ of the downstroke) after
takeoff in video 2. (J-L) The second downstroke ( $36 \%$ to $64 \%$ of the downstroke) after takeoff in video 2. For both videos, the first downstroke was $28.6 \mu \mathrm{~s}$ and the second downstroke was $24.2 \mu \mathrm{~s}$. The leftmost plots are the same as Fig. 3D for their respective downstrokes. The middle plots are the same as Fig. 3F for their respective downstrokes. The rightmost plots are the same as Fig. 3I for their respective downstrokes. Erroneous lines are due to occasional local stripe mismatching occurring in the image processing steps, but overall the automated 3D reconstruction is robust and accurate.

Table S1. Variable definitions. Brackets indicate the size of matrices [rows x columns] or the cell number of matrices [row, column], parentheses are used for coordinates, inequality signs are used for vectors $<\mathrm{x}, \mathrm{y}, \mathrm{z}>$, and 'pix' stands for pixels.

| Variable | Units | Description/Equation |
| :---: | :---: | :---: |
| Planes - known (projected) |  |  |
| $\left\langle A_{K}, B_{K}, C_{K}\right\rangle$ | - | Unit normal of $\mathrm{K}^{\text {th }}$ vertical known plane |
| M | - | Number of vertical known planes |
| $\left\langle D_{L}, E_{L}, F_{L}\right\rangle$ | - | Unit normal of $\mathrm{L}^{\text {th }}$ horizontal known plane |
| $N$ | - | Number of horizontal known planes |
| Planes - unknown (camera image) |  |  |
| $\left\langle a_{k}, b_{k}, c_{k}\right\rangle$ | - | Unit normal of $\mathrm{k}^{\text {th }}$ vertical unknown plane |
| $m$ | - | Number of vertical unknown planes |
| $\left\langle d_{l, ~}, e_{l} f_{l}\right\rangle$ | - | Unit normal of $1^{\text {th }}$ horizontal unknown plane |
| $n$ | - | Number of horizontal unknown planes |
| Points |  |  |
| $\mathbf{P}_{\text {c }}$ | $\mathrm{pix}_{\text {cam }}$ | 2D homogeneous camera coordinates [ $3 \times 1$ ] |
| $\mathbf{P}_{\mathrm{p}}$ | $\mathrm{pix}_{\mathrm{proj}}$ | 2D homogeneous projector coordinates [ $3 \times 1$ ] |
| $\mathbf{P}_{\text {w }}$ | mm | 3 D world coordinates [ $3 \times 1$ ] |
| ( $x_{k l}, y_{k l}$ ) | $\mathrm{pix}_{\text {cam }}$ | 2 D intersection of $\mathrm{k}^{\text {th }}$ vertical and $\mathrm{l}^{\text {th }}$ horizontal planes |
| Calibration |  |  |
| $\mathbf{K}_{\text {c }}$ | $\mathrm{pix}_{\text {cam }}$ | Internal camera calibration matrix [ $3 \times 3$ ] |
| $\alpha_{c}$ | pix ${ }_{\text {cam }}$ | Camera focal length (x), $\alpha_{c}=K_{c}[1,1]$ |
| $\beta_{c}$ | pix ${ }_{\text {cam }}$ | Camera focal length (y), $\beta_{c}=K_{c}[2,2]$ |
| $u_{0 c}$ | pix ${ }_{\text {cam }}$ | Camera principal point (x), $u_{o c}=K_{c}[1,3]$ |
| $v_{0 c}$ | pix $_{\text {cam }}$ | Camera principal point (y), $v_{o c}=K_{c}[2,3]$ |
| $\mathbf{K}_{\mathrm{p}}$ | $\mathrm{pix}_{\text {proj }}$ | Internal projector calibration matrix [ $3 \times 3$ ] |
| $\alpha_{p}$ | $\mathrm{pix}_{\text {proj }}$ | Projector focal length (x), $\alpha_{p}=K_{p}[1,1]$ |
| $\beta_{p}$ | pix ${ }_{\text {proj }}$ | Projector focal length (y), $\beta_{p}=K_{p}[2,2]$ |
| $u_{0}{ }_{0}$ | $\mathrm{pix}_{\text {proj }}$ | Projector principal point (x), $u_{o_{p}}=K_{p}[1,3]$ |
| $v_{0}{ }_{p}$ | pix ${ }_{\text {proj }}$ | Projector principal point (y), $v_{o p}=K_{p}[2,3]$ |
| R | - | Rotation matrix: camera to projector [ $3 \times 3$ ] |
| T | mm | Translation matrix: camera to projector [ $3 \times 1$ ] |
| Singular Value Decomposition |  |  |
| X | - | Contains $m$ vertical ( $a, b, c$ ) and $n$ horizontal planes ( $d, e, f$ ) $[m+n \times 1]$ |
| $\mathbf{X}_{\mathrm{p}}$ | - | A particular solution of $X[m+n \times 1]$ |
| $\mathbf{X}_{\text {h }}$ | - | The homogeneous solution of $X[m+n \times 1]$ |
| $p$ | - | Tuning parameter to choose a particular solution for $X$ |
| B | - | Constant matrix [ $2(m+n)+(\#$ intersections) $\times 1]$ |
| M | - | Constant matrix [2 $(m+n)+(\#$ intersections $) \times m+n]=[i \times j]$ |
| U | - | [ $i \times i$ ] unitary matrix of $\operatorname{SVD}(M)$ |
| $\sigma_{s}$ | - | $s^{\text {th }}$ singular value $M$ of $j$ values |
| V | - | $[j \times j]$ unitary matrix of $\operatorname{SVD}(M)$ |

