

Script 1. SOURCE CODE FOR PROC MODEL IN SAS 9.4

```
data WORK.IMPORT;
input X1 X2 Y;
datalines;

/*1 STRAIGHT LINE (NO INTERCEPT) FITTED TO RAW DATA; NORMAL, HOMOSCEDASTIC
ERROR, WITH INTERACTION */
proc model data=WORK.IMPORT method=Marquardt PRL=both;
parms b=1 c=0 d=0;
Y = b*X1 + c*X2 + d*X1*X2;
h.Y = sigma**2;
fit Y/ FIML normal white breusch=(1 X1) out = output1 outpredict outresid;
run;

/*2 STRAIGHT LINE (NO INTERCEPT) FITTED TO RAW DATA; NORMAL, HOMOSCEDASTIC
ERROR, WITH NO INTERACTION */
proc model data=WORK.IMPORT method=Marquardt PRL=both;
parms b=1 c=0;
Y = b*X1 + c*X2;
h.Y = sigma**2;
fit Y/ FIML normal white breusch=(1 X1) out = output2 outpredict outresid;
run;

/* 3 STRAIGHT LINE (NO INTERCEPT) FITTED TO RAW DATA; NORMAL, HETEROSCEDASTIC
ERROR, WITH INTERACTION */
proc model data=WORK.IMPORT method=Marquardt PRL=both;
parms b=1 c=0 d=0;
Y = b*X1 + c*X2 + d*X1*X2;
h.Y = sigma**2 * (X1**(2*alpha));
fit Y/ FIML normal white breusch=(1 X1) out = output3 outpredict outresid;
run;

/* 4 STRAIGHT LINE (NO INTERCEPT) FITTED TO RAW DATA; NORMAL, HETEROSCEDASTIC
ERROR, WITH NO INTERACTION */
proc model data=WORK.IMPORT method=Marquardt PRL=both;
parms b=1 c=0;
Y = b*X1 + c*X2;
h.Y = sigma**2 * (X1**(2*alpha));
fit Y/ FIML normal white breusch=(1 X1) out = output4 outpredict outresid;
run;

/* 5 STRAIGHT LINE (NO INTERCEPT) FITTED TO RAW DATA; LOGNORMAL,
HETEROSCEDASTIC ERROR, WITH INTERACTION */
proc model data=WORK.IMPORT method=Marquardt PRL=both;
parms b=1 c=0 d=0;
Y = b*X1 + c*X2 + d*X1*X2;
resid.Y = log (actual.Y/pred.Y);
fit Y/ FIML normal white breusch=(1 X1) out = output5 outpredict outresid;
run;

/* 6 STRAIGHT LINE (NO INTERCEPT) FITTED TO RAW DATA; LOGNORMAL,
HETEROSCEDASTIC ERROR, WITH NO INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;
parms b=1 c=0;
Y = b*X1 + c*X2;
resid.Y = log (actual.Y/pred.Y);
fit Y/ FIML normal white breusch=(1 X1) out = output6 outpredict outresid;
run;
```

```
/* 7 STRAIGHT LINE (INTERCEPT) FITTED TO RAW DATA; NORMAL, HOMOSCEDASTIC ERROR,
WITH INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;
parms b=1 c=0 d=0 y0=0;
Y = b*X1 + c*X2 + d*X1*X2 + y0;
h.Y = sigma**2;
fit Y/ FIML normal white breusch=(1 X1) out = output7 outpredict outresid;
run;
```

```
/* 8 STRAIGHT LINE (INTERCEPT) FITTED TO RAW DATA; NORMAL, HOMOSCEDASTIC ERROR,
WITH NO INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;
parms b=1 c=0 y0=0;
Y = b*X1 + c*X2 + y0;
h.Y = sigma**2;
fit Y/ FIML normal white breusch=(1 X1) out = output8 outpredict outresid;
run;
```

```
/* 9 STRAIGHT LINE (INTERCEPT) FITTED TO RAW DATA; NORMAL, HETEROSCEDASTIC
ERROR, WITH INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;
parms b=1 c=0 d=0 y0=0;
Y = b*X1 + c*X2 + d*X1*X2 + y0;
h.Y = sigma**2 * (X1**(2*alpha));
fit Y/ FIML normal white breusch=(1 X1) out = output9 outpredict outresid;
run;
```

```
/* 10 STRAIGHT LINE (INTERCEPT) FITTED TO RAW DATA; NORMAL, HETEROSCEDASTIC
ERROR, WITH NO INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;
parms b=1 c=0 y0=0;
Y = b*X1 + c*X2 + y0;
h.Y = sigma**2 * (X1**(2*alpha));
fit Y/ FIML normal white breusch=(1 X1) out = output10 outpredict outresid;
run;
```

```
/* 11 STRAIGHT LINE (INTERCEPT) FITTED TO RAW DATA; LOGNORMAL, HETEROSCEDASTIC
ERROR, WITH INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;
parms b=1 c=0 d=0 y0=0;
Y = b*X1 + c*X2 + d*X1*X2 + y0;
resid.Y = log (actual.Y/pred.Y);
fit Y/ FIML normal white breusch=(1 X1) out = output11 outpredict outresid;
run;
```

```
/* 12 STRAIGHT LINE (INTERCEPT) FITTED TO RAW DATA; LOGNORMAL, HETEROSCEDASTIC  
ERROR, WITH NO INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;  
parms b=1 c=0 y0=0;  
Y = b*X1 + c*X2 + y0;  
resid.Y = log (actual.Y/pred.Y);  
fit Y/ FIML normal white breusch=(1 X1) out = output12 outpredict outresid;  
run;
```

```
/* 13 POWER FUNCTION (NO INTERCEPT) FITTED TO RAW DATA; NORMAL, HOMOSCEDASTIC  
ERROR, WITH INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;  
parms a=1 b=1 c=0 d=0;  
Y = a * X1**(b+d*X2)*exp(c*X2);  
h.Y = sigma**2;  
fit Y/ FIML normal white breusch=(1 X1) out = output13 outpredict outresid;  
run;
```

```
/* 14 POWER FUNCTION (NO INTERCEPT) FITTED TO RAW DATA; NORMAL, HOMOSCEDASTIC  
ERROR, WITH NO INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;  
parms a=1 b=1 c=0;  
Y = a * X1**b*exp(c*X2);  
h.Y = sigma**2;  
fit Y/ FIML normal white breusch=(1 X1) out = output14 outpredict outresid;  
run;
```

```
/* 15 POWER FUNCTION (NO INTERCEPT) FITTED TO RAW DATA; NORMAL,  
HETEROSCEDASTIC ERROR, WITH INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;  
parms a=1 b=1 c=0 d=0;  
Y = a * X1**(b+d*X2)*exp(c*X2);  
h.Y = sigma**2 * (X1**(2*alpha));  
fit Y/ FIML normal white breusch=(1 X1) out = output15 outpredict outresid;  
run;
```

```
/* 16 POWER FUNCTION (NO INTERCEPT) FITTED TO RAW DATA; NORMAL,  
HETEROSCEDASTIC ERROR, WITH NO INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;  
parms a=1 b=1 c=0;  
Y = a * X1**b*exp(c*X2);  
h.Y = sigma**2 * (X1**(2*alpha));  
fit Y/ FIML normal white breusch=(1 X1) out = output16 outpredict outresid;  
run;
```

```
/* 17 POWER FUNCTION (NO INTERCEPT) FITTED TO RAW DATA; LOGNORMAL,  
HETEROSCEDASTIC ERROR, WITH INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;  
parms a=1 b=1 c=0 d=0;  
Y = a * X1**(b+d*X2)*exp(c*X2);  
resid.Y = log (actual.Y/pred.Y);  
fit Y/ FIML normal white breusch=(1 X1) out = output17 outpredict outresid;  
run;
```

```
/* 18 POWER FUNCTION (NO INTERCEPT) FITTED TO RAW DATA; LOGNORMAL,  
HETEROSCEDASTIC ERROR, WITH NO INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;  
parms a=1 b=1 c=0;  
Y = a * X1**b*exp(c*X2);  
resid.Y = log (actual.Y/pred.Y);  
fit Y/ FIML normal white breusch=(1 X1) out = output18 outpredict outresid;  
run;
```

```
/* 19 POWER FUNCTION (INTERCEPT) FITTED TO RAW DATA; NORMAL, HOMOSCEDASTIC  
ERROR, WITH INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;  
parms a=1 b=1 c=0 d=0 y0=0;  
Y = a * X1**(b+d*X2)*exp(c*X2) + y0;  
h.Y = sigma**2;  
fit Y/ FIML normal white breusch=(1 X1) out = output19 outpredict outresid;  
run;
```

```
/* 20 POWER FUNCTION (INTERCEPT) FITTED TO RAW DATA; NORMAL, HOMOSCEDASTIC  
ERROR, WITH NO INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;  
parms a=1 b=1 c=0 y0=0;  
Y = a * X1**b*exp(c*X2) + y0;  
h.Y = sigma**2;  
fit Y/ FIML normal white breusch=(1 X1) out = output20 outpredict outresid;  
run;
```

```
/* 21 POWER FUNCTION (INTERCEPT) FITTED TO RAW DATA; NORMAL, HETEROSCEDASTIC  
ERROR, WITH INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;  
parms a=1 b=1 c=0 d=0 y0=0;  
Y = a * X1**(b+d*X2)*exp(c*X2) + y0;  
h.Y = sigma**2 * (X1**(2*alpha));  
fit Y/ FIML normal white breusch=(1 X1) out = output21 outpredict outresid;  
run;
```

```
/* 22 POWER FUNCTION (INTERCEPT) FITTED TO RAW DATA; NORMAL, HETEROSCEDASTIC  
ERROR, WITH NO INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;  
parms a=1 b=1 c=0 y0=0;  
Y = a * X1**b*exp(c*X2) + y0;  
h.Y = sigma**2 * (X1**(2*alpha));  
fit Y/ FIML normal white breusch=(1 X1) out = output22 outpredict outresid;  
run;
```

```
/* 23 POWER FUNCTION (INTERCEPT) FITTED TO RAW DATA; LOGNORMAL,  
HETEROSCEDASTIC ERROR, WITH INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;  
parms a=1 b=1 c=0 d=0 y0=0;  
Y = a * X1**(b+d*X2)*exp(c*X2) + y0;  
resid.Y = log (actual.Y/pred.Y);
```

```
fit Y/ FIML normal white breusch=(1 X1) out = output23 outpredict outresid;  
run;
```

```
/* 24 POWER FUNCTION (INTERCEPT) FITTED TO RAW DATA; LOGNORMAL,  
HETEROSCEDASTIC ERROR, WITH NO INTERACTION */
```

```
proc model data=WORK.IMPORT method=Marquardt PRL=both;
```

```
parms a=1 b=1 c=0 y0=0;
```

```
Y = a * X1**(b)*exp(c*X2) + y0;
```

```
resid.Y = log (actual.Y/pred.Y);
```

```
fit Y/ FIML normal white breusch=(1 X1) out = output24 outpredict outresid;
```

```
run;
```

```
/* IF NONLINEAR MODELS FAIL TO CONVERGE, TRY INCREASING THE NUMBER  
OF ITERATIONS AND/OR RELAXING THE CONVERGENCE CRITERION, e.g.,  
  iter=200 converge=0.01  AND TRY CHANGING START VALUES FOR THE  
PARAMETERS. IF THE VALUE FOR c WAS NOT SIGNIFICANT, THEN THE  
PARAMETER c WAS DROPPED FROM THE MODEL AND THE MODEL WITH NO X2  
TERM WAS USED TO ANALYSIS. */
```

Table S1. Statistical models fitted to untransformed data for scalings.

[Click here to Download Table S1](#)