

Figure S1. Graphical demonstration of rule 2: $V\left(r ; f_{1} \ldots f_{n}\right) \leq V\left(r ; f_{1} \ldots\right.$ $f_{n}, f_{n+1}$ ) which states that the vividness in a colour solid either increases or stays the same when an extra dimension (spectral photoreceptor class) is added.
Left to right is the component of the original solid parallel to the vector from the centre of the solid, $O$, to the colour $A$. The corresponding boundary colour is labeled $B$. The upwards direction shows the added dimension. The value in this dimension does not affect the value in the original dimension, so the position in this new space $C$ is on a vertical line passing through $A$. The vividness in the original solid is $\overline{O A} / \overline{O B}$ and becomes $\overline{O C} / \overline{O D}$. We can then use a long established geometric fact (it is proposition 2 in Book VI of Euclid's Elements) that dividing two edges of a triangle by the same ratio gives points that lie on a line parallel with the third edge (and viceversa). The line through $B$ and parallel to the one passing though $C$ and $A$ is vertical by virtue of this proposition. This line contains point $E$, which is the position that $D$ would need to be in for equal vividness. If $D$ is closer to $O$ than $E$ then the vividness is greater. As $B$ is a boundary point, the convexity of the colour solid means that there are no points to be found further to the right on this diagram, demonstrating the proposition. In summary $V\left(r ; s_{1} \ldots s_{n}, s_{n+1}\right)=\overline{O C} / \overline{O D} \geq \overline{O C} / \overline{O E}=\overline{O A} / \overline{O B}=V\left(r ; s_{1} \ldots s_{n}\right)$


Figure S2. Graphical demonstration of rule 3: $V\left(k r_{1}+(1-k) r_{2} ; f_{1} \ldots f_{n}\right)$ $\leq \max \left\{V\left(r_{1} ; f_{1} \ldots f_{n}\right), V\left(r_{2}, f_{1} \ldots f_{n}\right)\right\}$
This can be be interpreted in terms of the following diagram of the plane containing the colours of $r_{1}$ and $r_{2}$ and the centre of the solid: $O$ is the centre of the solid. $A$ and $B$ correspond to the reflectance spectra $r_{1}$ and $r_{2}$ with the diagram drawn where $B$ is the more vivid of the two. $C$ is some convex combination of $A$ and $B$, i.e. a point on the line between them. From the same rule used in the previous section (Elements VI, 2) we know that $\overline{O B} / \overline{O X}=\overline{O B^{\prime}} / \overline{O X^{\prime}}=\overline{O B^{\prime \prime}} / \overline{O X^{\prime \prime}}$. The ratio $\overline{O A} / \overline{O X^{\prime}}$ is smaller than $\overline{O B^{\prime}} / \overline{O X^{\prime}}$ as we have chosen $B$ to be the more vivid of the two. The diagram shows that as $\overline{O A}$ is (weakly) shorter than $\overline{O B^{\prime}}$, then $\overline{O C}$ is (weakly) shorter than $\overline{O B^{\prime \prime}}$ Furthermore, from the convexity of the colour solid it follows that $\overline{O X^{\prime \prime}}$ is weakly shorter than $\overline{O Y}$. So, we can say that $\overline{O B^{\prime \prime}} / \overline{O X^{\prime \prime}} \geq \overline{O C} / \overline{O X^{\prime \prime}} \geq \overline{O C} / \overline{O Y}$. Which is the proposition to be demonstrated as $\overline{O B^{\prime \prime}} / \overline{O X^{\prime \prime}}$ is $\max \left\{V\left(r_{1} ; x_{1} \ldots x_{n}\right), V\left(r_{2}, x_{1} \ldots x_{n}\right)\right\}$ and $\overline{O C} / \overline{O Y}$ is $V\left(k r_{1}+(1-k) r_{2} ; x_{1} \ldots x_{n}\right)$.

